TILDA - Symposium & Workshop on Industrial LES & DNS, Toulouse, France, November 21-23, 2016

TILDA

Revisiting the spectral analysis for high-order spectral discontinuous methods

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October 27, 2016

Abstract: The spectral analysis is a basic tool to characterise the behaviour of any convection scheme. By nature, the solution projected onto the Fourier basis enables to estimate the dissipation and the dispersion associated with the spatial discretisation of the hyperbolic linear problem. Such an approach is the prerequisite to define the associated Points Per Wavelength -PPW- for capturing the solution wavelength with a given mesh refinement. In this work, we wish to revisit such analysis focusing attention on the Spectral Difference method [1, 2].

First, using a space/time analysis, a new way to address the specific spectral behaviour of the scheme is introduced for wavenumbers in $[0, \pi]$, following the matrix power method. For wavenumbers above π, an aliasing phenomenon always occurs but it is now possible to understand and to control the aliasing of the signal. It is shown that aliasing depends on the polynomial degree and on the number of time steps. A new way to define dissipation and dispersion is introduced and applied to wavenumbers larger than π. Since the new criteria recover the previous results for wavenumbers below π, the new approach is an extension of all the previous ones for analysing dissipation and dispersion.

To conclude, the proposed analysis revisits the spectral analysis first introduced by Van den Abeele [3] and gives new explanations regarding the spectral behaviour. The new spectral analysis leads to results that differ from the ones published in previous studies but the mathematical framework introduced in the presentation is well-posed and confirmed by numerical simulations.

Keywords: space-time spectral analysis, spectral discontinuous, Nyquist theorem, aliasing, matrix power method.

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